Credibility

Presented by

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Consulting Actuary

Milliman

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Annual Meeting

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Credibility

"When the facts change, I change my opinion

What do you do, sir?"

- J. M. Keynes

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Agenda

- Current actuarial practice
- A brief history of Bayesian statistics
- Applications
- References

Current Practice

Common applications

- Health
  - Medicare Advantage CMS bidding
- Property & Casualty
  - Bornhuetter-Ferguson loss reserving method
- Life
  - Mortality assumption setting

Current Practice

ASOP 25: Credibility Procedures

- December 2013
- Update from October 1996
- Added:
  - Definitions: Credibility Procedure, Risk Characteristics, Risk Classification System
  - Disclosures: ASOP 41, *Actuarial Communication*
  - Current Practices: Modern statistical techniques (GLM)

Current Practice

ASOP 25: Current Practices

- Classical (Limited Fluctuation)
- Empirical ("without reference to the underlying distribution")
- Bayesian (example: Bühlmann, or Greatest Accuracy Credibility)
- Emerging Practice Involving Statistical Models

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Current Practice

Classical, Limited Fluctuation ("frequentist")

- Assume underlying probability distribution
- Estimate number of claims ($ premium) such that:

  P(subject loss experience is within specified range) = p
- Longley-Cook ('62) uses the Poisson distribution to determine p's for given specified ranges

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Current Practice

Classical, Limited Fluctuation ("frequentist")

- Example: estimating AA 80 mortality rates for a certain pricing cell
- Want to be 90% sure that the mortality rate in my data falls within (+/-) 10% from the underlying expected rate
- How many claims do I need in my data set ("subject experience") ?
  - Answer: 271 claims

Current Practice

Classical, Limited Fluctuation ("frequentist")

- Table of required counts for given ranges, probabilities
- *"Full Credibility"*

Max departure from expected	Probability of meeting test		
	99%	95%	90%
+/- 2.5%	10,623	6,147	4,326
+/- 5.0%	2,656	1,537	1,082
+/- 7.5%	1,180	683	481
+/- 10%	664	384	271

Current Practice

Classical, Limited Fluctuation (“frequentist”)

For partial probability, apply the square root rule:

Credibility Factor = $\sqrt{\frac{count\ of\ claims}{full\ credibility\ standard}}$

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Current Practice

- Bayesian Credibility
  - An alternative to the limited fluctuation (classical, frequentist) approach …..

…. with a controversial history

A Brief History

The Reverend Thomas Bayes

Pierre Simon Laplace

A Brief History

Bayes Theorem

aka

- "the probability of causes"
- "inverse probabilities"
- "Bayes-Laplace"
- (and a few hundred other variations of "Bayes-OtherSurname")

Historical Influences:

- Formalized by Richard Price (the "Bayes-Price" theorem)
- Universalized by Pierre Simon Laplace

A Brief History

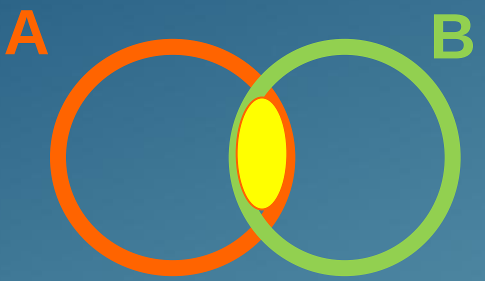
We recognize this as:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

or

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

graphically:



$A \cap B = P(A|B)\,P(B)$

$= P(B|A)\,P(A)$

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A Brief History

$$P(cause|effect) = \frac{P(effect|cause)}{\sum_{causes} P(effect|cause)}$$

- Controversy: the assumption of equal priors (i.e. should all initial causes be assumed equally likely?)
  - Theological component
- Side-lined Bayes theorem for decades

A Brief History

- After Laplace, a long remission
  - Even Laplace favored frequentist statistics towards end of career
- Bayes resurrected in the 20th century
  - Though not without a fight
- Prior distribution assumptions: not a problem for actuaries!

A Brief History

Some serious results for Bayes:

- Alan Turing and the cracking of the Enigma machine
  - Responsible for saving thousands of lives, ending WWII earlier
  - Finding the Enigma setting which had encoded a particular message – a classic problem in the inverse probability of causes
- The author of 12 of the anonymous *The Federalist* Papers
  - The odds – using Bayesian methods – were 'satisfyingly fat' that Madison wrote all 12 of them

A Brief History

Some serious results for Bayes:

- Finding lost hydrogen bombs during the cold war
- Nate Silver's prediction of the 2012 presidential election
- Bayesian Statistics using Gibbs Sampling (BUGS)
- Markov Chain Monte Carlo (MCMC) processes

A Brief History

Bayes and actuarial science

- Albert Wurts Whitney, Berkeley
- Casualty Actuary Society, 1918
- Developed simplified Bayes approach
  - $Z = P / (P+K)$
  - Quipped an actuary: “of course [the formula] is not so great a discovery as $E=mc^2$ nor as unalterably true, but it has made life much easier for insurance men for generations”

A Brief History

Bayes and actuarial science

- Arthur L Bailey
  - 1947: Chief Actuary of NY State Insurance Department
  - 1950: VP, Kemper Insurance Group (Chicago)
  - Paper: *Credibility Procedures: Laplace's Generalization of Bayes' Rule and the Combination of Collateral [prior] Knowledge with Observed Data*
- Hans Bühlmann
  - Produced a general theorem of Bayesian credibility
  - "Why should a confidence interval that, by definition, includes the true value with a probability of less than one, guarantee full credibility?"

$$Z = \frac{n}{n + {}^{v}/_{a}}$$

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A Brief History

Bayes and actuarial science

- Thomas Herzog
  - Introduction to Credibility Theory
  - AAA Practice Note
- Stuart Klugman
  - Applied modern computing techniques to Bayes re: workers comp
  - Dec 2009 Survey:

The major conclusion from this survey of 190 US insurers is that credibility theory is not widely adopted among surveyed actuaries at United States life and annuity carrier to date in managing mortality-, lapse- and expense- related risks

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Applications

A General Form for Credibility (both limited fluctuation and Bayes)

Estimate = Z x R + (1 – Z) x H

Z = Credibility factor

R = mean of current observations

H = prior mean

To calculate Z ….

Applications

1. Mortality Example

(from *Credibility Theory Practices* report, Klugman et al, 2009)

- Nonsmoker mortality
- 10 surveyed companies
- Expected amounts from A/E based on 2001 VBT

- Study used company data to calculate statistics (moments) of the estimators, rather than assume distribution (such as in Longley-Cook)

Applications

1. Mortality Example

(from *Credibility Theory Practices* report, Klugman et al, 2009)

Results:

- Overall “industry” mortality A/E by count = 83.8%

- Company B:
  - Company A/E ratio = 125.6%
  - 1,038 deaths

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Applications

1. Mortality Example

(from *Credibility Theory Practices* report, Klugman et al, 2009)

Results:

- Company B Credibility Factors (1,038 deaths):
  - Longley-Cook: = $\sqrt{{}^{1,038}/_{1,082}} \cong 98\%$
  - Klugman Study
    - Limited Fluctuation: 83.0%
    - Buhlmann Empirical Bayes: 94.5%

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Applications

1. Mortality Example

(from *Credibility Theory Practices* report, Klugman et al, 2009)

Results:

Company B mortality A/E by amount

Method	Industry A/E	Company A/E	Credibility Factor (Z)	Final A/E
Longley-Cook	83.8%	125.6%	97.9%	**124.7%**
Limited Fluctuation	83.8%	125.6%	83.0%	**118.5%**
Bühlmann Empirical Bayes	83.8%	125.6%	94.5%	**123.3%**

Applications

2. LTC Incidence Rates

(inspired by *Credibility Practice Note*, 2008)

- Incidence Rates determined for use in pricing a new product
- What rate to use for a 45 yr old
- Industry data (for example, collected via Milliman's Long-Term Care Guidelines)
- Company data

Applications

2. LTC Incidence Rates
(inspired by *Credibility Practice Note*, 2008)

- Industry Data
  - 2000+ overall claims (i.e. “fully credible” via Longley-Cook)
  - Suggests age 45 incidence rate = 0.4%

- Company data
  - Age 45 is young for LTC
  - New company
  - Only 8 claims for an exposure of 5000 life-years
  - Suggests incidence rate of 0.16%

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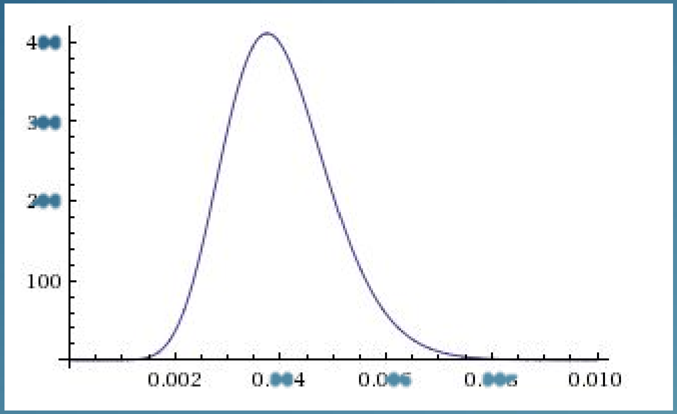
Applications

2. LTC Incidence Rates

(inspired by *Credibility Practice Note*, 2008)

- Bayes Approach
  - Claim incidence is binomial (makes sense re: incidence as 1 or 0 event)
  - Assume prior distribution: Beta
    - Easy to work with
    - Appropriate shape
    - (it's complicated to make up your own example)
  - Z = 55.6%
  - Estimated Incidence: 0.27%



Applications

2. LTC Incidence Rates

(inspired by *Credibility Practice Note*, 2008)

– Limited Fluctuation
  - 8 claims
  - Want 90% chance of being within 5% (+/-) of true mean
  - Z = $\sqrt{8/_{1,082}} \cong 8.6\%$
  - Estimated Incidence: 0.38%

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Applications

2. LTC Incidence Rates

(inspired by *Credibility Practice Note*, 2008)

Incidence Estimate, age 45

Method	Industry Incidence	Company Incidence	Credibility Factor (Z)	Final Incidence
Bayes / Bühlmann	0.4%	0.16%	55.6%	**0.27%**
Limited Fluctuation	0.4%	0.16%	8.6%	**0.38%**

Applications

Summary

- Potential for large differences in results
- The actuary must be prepared to justify
- Understand track record for past, similar estimates
- Don’t:
  - Use a particular credibility method strictly because that’s how it was done before, or
  - Select a credibility method based on ease of use
- Do: take on the challenge of assigning reasonable credibility estimates based on best practices

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Questions?

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