# Real World Applications of Stochastic Models

June 24, 2005

Nathan Hardiman

Rebecca Scotchie

JEFFERSON PILOT
FINANCIAL

TOWERS
PERRIN
TILLINGHAST

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Seems like every actuarial publication has articles on requirements involving stochastic models

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SOCIETY OF ACTUARIES

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the actuary

The newsletter of the
Society of Actuaries

Principles-based regulation of variable annuities
Proposals now at pivotal stage

by Max J. Rudolph, FSA, CFA, MAAA

"This project takes another step down an evolutionary path that includes cash flow testing, regulation XXX and other reserve requirements that require multiple scenario testing and company specific data to support formula reserves."

## Managing the risks from variable annuities— the next phase

by Phil Bieluch and Hubert Mueller

Variable annuities typically contain guarantees that expose the carrier to risks of the equities markets. These annuities are either payable on... [text truncated]

"… the utilization of the GLBs is much tougher to predict since it is driven by policyholder behavior. Experience on the utilization of these riders is still limited and should be analyzed under a variety of market scenarios."

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## The Financial Reporter

## Rethinking Embedded Value:
### The Stochastic Modeling Revolution

Carol A. Marler and Vincent Y. Tsang

In the United States, all publicly traded insurance companies prepare at least three sets of financial statements: statutory, GAAP and tax. These three sets of financials are prepared for different purposes and do...

Australia, as well as to most of E... extent, the concept is also sprea... United States, as many companies... their European parent companies. W...

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## RISKS AND REWARDS

### Deflators—The Solution to a Stochastic Conundrum?

by Don Wilson

Stochastic modeling of life insurance products has become increasingly important over the last few years. The complex nature of the guarantees that exist in many products has generally required the use of a Monte Carlo approach, involving the calculations being performed repeatedly for each scenario, potentially many hundreds or thousands of times.

**LEHMAN BROTHERS**

EQUITY RESEARCH

June 18, 2004

North America

Financial Services

**Insurance/Life**

Insurance/Life

Industry Update

Eric N. Berg, CPA, CFA

1.212.526.2805

700 Concerned Life-Insurance Actuaries

eberg@lehman.com

**Sector View:**

"Life companies that formerly used a so-called 'factor' approach to establish reserves…will now have to become experts in stochastic modeling."

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## Discussion Topics

- Why use stochastic models?
- Stochastic modeling mini-tutorial
- Value-added applications of stochastic modeling

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## Why use Stochastic Models?

- Valuable means of measuring and analyzing risks of our business
  - Most beneficial for risks with low frequency and high severity
- Maximize profitability for a given level of risk
- Aid in making business decisions

## Why use Stochastic Models?

- Regulatory requirements driving the use of Stochastic Models:
  - Move toward principles-based regulation
  - Proposal for C-3 Phase II RBC
  - Proposed AG VACARVM
- Regulation suggests use of stochastic modeling
  - GAAP SOP 03-1

## Coming Soon…

- If stochastic modeling is not currently being done, it will likely be required at some point (probably sooner rather than later).
- Building these models is:
  - Costly
  - Time consuming
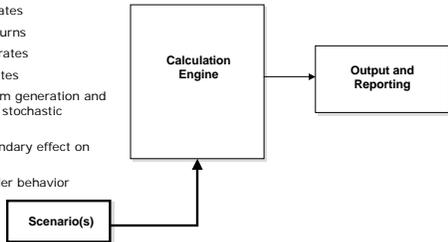  - Huge initial effort

## Coming Soon…

- Actuaries need to help upper management avoid the perspective of modeling as an expensive regulatory exercise.

- Show value of modeling in better understanding, pricing, and management of the business.

## We have all been modeling for most of our career

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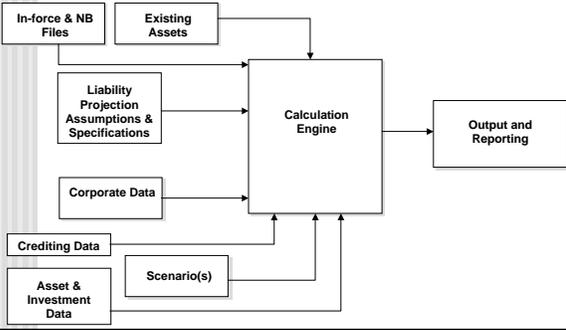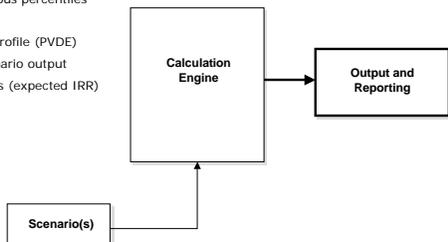
## What makes a model "stochastic"?

- Projecting model repeatedly varying one or more items, e.g.:
  - Interest rates
  - Equity returns
  - Mortality rates
  - Default rates
- Involves random generation and sampling from a stochastic distribution
- May have secondary effect on another value
  - Policyholder behavior

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## What output is valuable from a "stochastic" model ?

- Range of results
- Results at various percentiles
- CTE measures
- Graph of risk profile (PVDE)
- Individual scenario output
- Average results (expected IRR)

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# Value Added Applications

- Develop risk/reward profile of new or existing products
- Assess effect of assumptions in extreme scenarios
  - Only as good as assumed relationship
    - E.g., assumed formula to model policyholder behavior
    - Two formulas thought to be equally credible could give different results. Which is correct?
  - Take care to not create a false sense of security

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ILLUSTRATIVE

**Risk Profile of Potential Product Designs**



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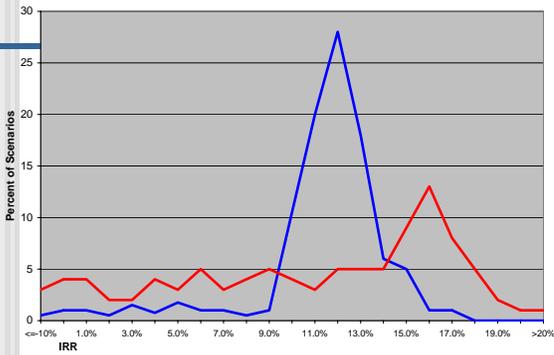
# Value Added Applications

- Calculate expected returns on new sales and existing blocks of business
- Test management strategies
  - Investment Strategy
  - Crediting Strategy

**Potential Investment Strategies**



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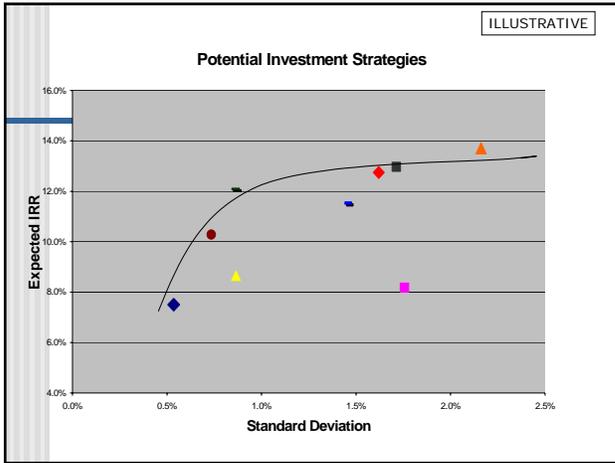
## Value Added Applications

- Assist in corporate planning/strategy
  - Analysis for rating agencies
  - Analysis for company analysts

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## More Complex Value Added Applications

- C-3 Phase II capital and AG VACARVM reserves
  - UL working group is exploring similar calculation
- Stochastic Mortality
- Nested stochastic applications
  - Projection of principles-based capital and reserves or GAAP SOP 03-1 reserves
    - Pricing or in-force projection
- GAAP SOP 03-1 Benefit reserves

## C-3 Phase II capital and AG VACARVM reserves

**The proposed requirements introduce a scenario-modeling approach to assessing regulatory capital and reserves**

C-3 Phase II capital equals CTE90 of the Total Asset Requirement (TAR) using prescribed C-3 scenarios, which meet certain calibration points, less statutory reserves actually held.

VACARVM reserves equal CTE65 of the TAR using the prescribed C-3 scenarios. For reserves, the TAR is before tax.

## C-3 Phase II capital and AG VACARVM reserves
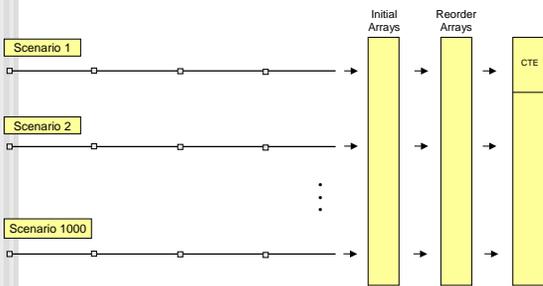


## Stochastic mortality

- Application of stochastic function to input mortality rate to make stochastic
  - Normal distribution
- Example:
  - Table rate d = 0.0027 for 45MNS
  - Apply $y \sim N(\mu, \sigma)$ where $\mu = 1$ and $\sigma = 0.1$
  - Therefore, mortality rate for each scenario equals d * y

# Nested Stochastic Applications

**Nested stochastic modeling is needed to determine capital and reserves for a pricing run which includes a projection of capital and reserves in the future.**



- Model Point 1, Scenario 1
- Model Point 2, Scenario 1
- Model Point 10, Scenario 1,000
- Experience Calculations
- Nested Calculations
- 30 yr projection

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# Nested Stochastic Applications



- Model Point 1, Scenario 1
- Experience Calculation
- C-3 RBC Calculations
- Initial Arrays
- Reorder Arrays
- CTE

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# GAAP SOP 03-1 benefit reserves

- Calculation of SOP 03-1 benefit reserves can be a stochastic modeling exercise at time 0.
- In a GAAP projection, there are several approaches available for calculating and reflecting SOP 03-1 benefit reserves

| Approaches | Description |
|---|---|
| **Use external projection system for generating benefit ratios and average benefit payment factors** | *Develop benefit ratios from external system and feed into projection, Calculate future benefit pmts. by applying factors to EGPs, or assessments* <br> (+) Simple to implement <br> (+) Leverage off of existing model <br> (-) External system needs to be consistent with DAC valuation system <br> (-) Does not accommodate stochastic modeling demands (discussed later) |
| **Build capabilities within system to perform stochastic projection at issue to develop benefit ratio and future average benefit payments** | *Explicitly perform a side calculation at issue stochastically generating benefit ratio and average future benefit payments* <br> (-) More difficult to implement <br> (-) Increased run time <br> (+) Promotes consistency, making system 'all-in-one' solution <br> (+) Sets the stage well for stochastic modeling demands |

# GAAP SOP 03-1 benefit ratio unlocking

- In a stochastic environment, the SOP benefit reserves would need to be unlocked to reflect actual collected assessment and benefit payments differing from expectations.
- This can be attacked in several ways:

| Approaches | Description |
|---|---|
| Use valuation date benefit ratio throughout projection | *Use approach developed for valuation system, leave constant throughout* |
| | (+) Simple to implement (created by val system) |
| | (+) Run time low |
| | (-) Accuracy questionable |
| Develop 'rules-of-thumb' for unlocking benefit ratios | *Perform analysis to determine a rule-of-thumb on unlocking benefit ratios, perhaps based on NAR, equity return change, other measures* |
| | (+) Simple to implement |
| | (+) Run time relatively low |
| | (-) Difficult to develop appropriate 'rules-of-thumb' |
| | (-) Accuracy questionable |
| Perform stochastic-on-stochastic projection | *Use system to perform a side stochastic projection at each unlocking period to explicitly re-evaluate and unlock benefit ratio* |
| | (-) More difficult to implement |
| | (-) Run time an issue |
| | (+) Most accurate, true-to-life approach |

# Questions?